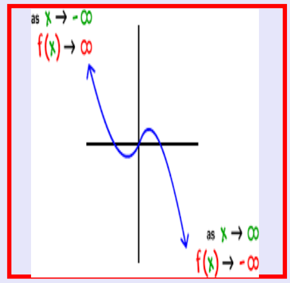
Math 245

Spring 2022

Lecture 40



$f(x) = \dfrac{x-4}{x-2}$

Rational Function $\quad \dfrac{\text{Polynomial}}{\text{Polynomial}}$

1) Domain: $x-2 \neq 0 \quad x \neq 2 \Rightarrow (-\infty, 2) \cup (2, \infty)$

2) Vertical Asymptote $x = 2$

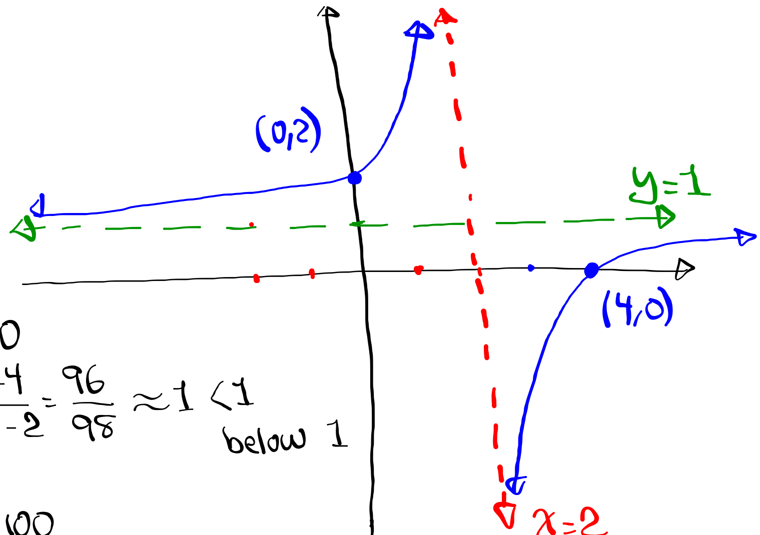
3) Y-Int $\to x = 0 \to f(0) = \dfrac{0-4}{0-2} = \dfrac{-4}{-2} = 2 \to$ Y-Int $(0,2)$

4) X-Int $\to y = 0 \to f(x) = 0 \to x - 4 = 0 \quad x = 4 \to$ X-Int $(4,0)$

5) Horizontal Asymptote:

when deg. of numerator = deg. of denominator

H.A. $\quad y = \dfrac{\text{Lead. coef. of Num.}}{\text{Lead. coef. of deno.}} \qquad y = \dfrac{1}{1} = 1$



If $x = 100$

$y = \frac{100-4}{100-2} = \frac{96}{98} \approx 1 < 1$

below 1

If $x = -100$

$y = \frac{-100-4}{-100-2} = \frac{-104}{-102} \approx 1 > 1$

Above 1

Range

$(-\infty, 1) \cup (1, \infty)$

$f(x) = \frac{x+4}{x^2-4}$ $\quad f(x) = \frac{x+4}{(x+2)(x-2)}$

1) Domain $\rightarrow$ Deno. $\neq 0$ $\quad x^2-4 \neq 0$ $\quad (x+2)(x-2) \neq 0$

$x+2 \neq 0 \rightarrow x \neq -2$

$x-2 \neq 0 \rightarrow x \neq 2$

-2 $\quad$ 2

$\rightarrow (-\infty,-2) \cup (-2,2) \cup (2,\infty)$

2) Vertical Asymptote $\rightarrow$ $x=-2$, $x=2$

3) Y-Int $\rightarrow x=0 \rightarrow f(0) = \frac{0+4}{0^2-4} = \frac{4}{-4} = -1$

$\rightarrow (0,-1)$

4) x-Int $\rightarrow y=0 \rightarrow f(x)=0 \rightarrow$ Num. $=0$

$x+4=0 \quad x=-4$

$\rightarrow (-4,0)$

5) Horizontal Asymptote:

when deg. of Numerator < deg. of denominator

H.A. $\quad y=0$

If $x=3$

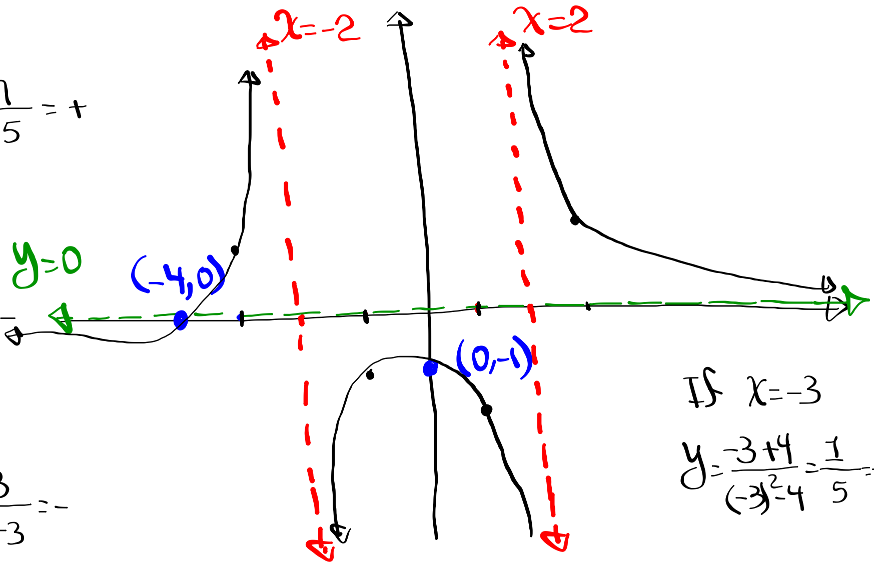
$y=\frac{3+4}{3^2-4}=\frac{7}{5}=+$

If $x=1$

$y=\frac{1+4}{1^2-4}=\frac{5}{-3}=-$

If $x=-1$

$y=\frac{-1+4}{(-1)^2-4}=\frac{3}{-3}=-$



If $x=-3$

$y=\frac{-3+4}{(-3)^2-4}=\frac{1}{5}=+$

$x=100$

$y=\frac{100+4}{100^2-4}\approx 0>0$ Above x-axis

$x=-100$

$y=\frac{-100+4}{(-100)^2-4}\approx 0<0$ below x-axis

$f(x)=\frac{x^2-4}{x}$

Domain: Deno $\neq 0$ → $x\neq 0$ → $(-\infty,0)\cup(0,\infty)$

Vertical Asymptote → $x=0$

Y-Int → $x=0$ but 0 is not in the domain

→ None

x-Int → $y=0$ → $f(x)=0$ → Num. $=0$ → $x^2-4=0$

$x=\pm 2$

→ $(2,0),(-2,0)$

when deg. of numerator > deg. of denominator

by only 1 → Slant Asymptote

To find it → do long division

$$\begin{array}{r|l} & \quad x \\ x & x^2 + 0x - 4 \\ & -(x^2 \qquad\quad) \\ \hline & \qquad\qquad -4 \end{array}$$

Slant Asymptote

→ $y=x$

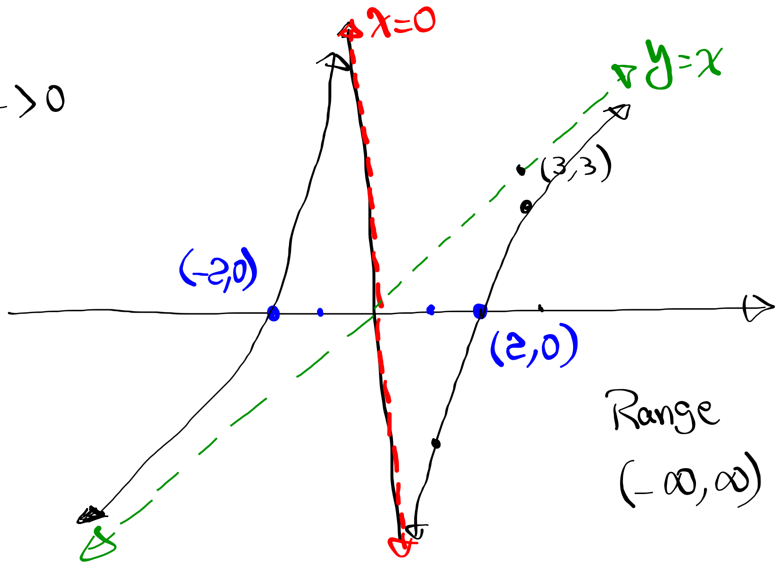
$x=3$

$y=\frac{3^2-4}{3}=\frac{5}{3}>0$

$\frac{5}{3}<3$

$x=1$

$y=\frac{1^2-4}{1}=-3$



Range

$(-\infty,\infty)$